CS269Q: Quantum Programming **Spring 2019**

Assignment #1

Due: 11:59pm on Mon., Apr 15 2019, by Gradescope (each answer on a separate page)

Problem 1. Measurements.

a. What is the probability that when measuring the first bit of a system in the state $(|00\rangle + |10\rangle + |11\rangle)/\sqrt{3}$ the outcome is 1?
What is the state of the system after such a measurement?

b. Show that the average value of the observable $M := X_1 Z_2$ (i.e., applying X to the first bit and Z to the second bit) for a two qubit system measured in the state $(|00\rangle + |11\rangle)/\sqrt{2}$ is zero.

Problem 2. Read Section 2.1.8 of Mike & Ike and then answer the following question. The rotation gates are defined by:

$$R_x(\theta) := e^{-i\theta X/2} = \cos(\theta/2)I - i\sin(\theta/2)X = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix} \tag{1}$$

$$R_y(\theta) := e^{-i\theta Y/2} = \cos(\theta/2)I - i\sin(\theta/2)Y = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix} \tag{2}$$

$$R_z(\theta) := e^{-i\theta Z/2} = \cos(\theta/2)I - i\sin(\theta/2)Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} \tag{3}$$

Show that for any real number x and matrix A such that $A^2 = I$:

$$e^{iAx} = \cos(x)I + i\sin(x)A.$$

Use this to verify the definitions of the rotation gates.

Problem 3. Superdense Coding. Read Section 2.3 of Mike & Ike to review an example of Superdense Coding using qubits.

a. Imagine that for a given Quantum Abstract Machine instance, half of the classical memory is available to Alice while half is available to Bob. Similarly half of the quantum memory is available to Alice while half is available to Bob. Write a Quil program that transmits two classical bits from Alice's half to Bob's using the superdense coding protocol.

b. Using Quil & pseudocode describe how this would extend to a Quil program that transmits an arbitrary number of classical bits from Alice to Bob using superdense coding. By Quil pseudocode can annotate your Quil code to describe the algorithm, e.g. by adding loops like

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for i in range(10): H 0
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Problem 4. Controlled Unitaries. Let U be an arbitrary unitary matrix on n qubits. A *controlled-U* operation applies U to those n qubits if a control qubit is 1 and does nothing otherwise. Thus controlled-U is $n+1$ qubit operation. *Hint* Thinking about what qubit index you choose to be the control may make this problem easier.

a. Let U be a single qubit gate. Construct the matrix for controlled-U.

b. Let U be a two qubit gate. Construct the matrix for controlled-U.

c. Write a program that takes an arbitrary n-qubit unitary (U) as input and returns a matrix for controlled-U.

Problem 5. A Quil Adder. Similarly to classical computation, it is common to represent numbers in quantum memory in registers of qubits. For example the binary number 00110 is represented by the quantum state $|00110\rangle$. In this problem you will construct Quil pseudocode to implement binary addition on quantum bits and registers.

a. (Quantum Half-Adder) Let x and y be two qubits. Construct a Quil program that performs the following operation:

$$|x, y, 0\rangle \mapsto |x, x \oplus y, xy\rangle$$

b. (Quantum Full-Adder) Let x and y be two qubits that are to be added together. Let c_{in} be another qubit that represents the incoming carry bit. Construct a Quil program that performs the following operation:

$$|x, y, c_{in}, 0\rangle \mapsto |x, y, x \oplus y \oplus c_{in}, c_{out}\rangle$$

c. (BONUS: Quantum Ripple Carry Adder) Let r and q be registers of qubits. Construct a pseudocode Quil program that adds two binary representations of numbers in quantum memory. Specifically, your Quil program should implement the transformation

$$|r, q\rangle \mapsto |r, r + q\rangle$$